PARAFAC Modeling/Estimation of Time-Varying Space-Time Multipath Channels

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Abstract—In this paper, we address the problem of multipath parameter estimation of time-varying space-time wireless channels based on PARAFAC modeling. We exploit the different varying rates in the structure of the multipath channel over multiple data-blocks to build a 3D tensor from the received signal. The proposed estimation methods make use of a training sequence that is periodically extended over multiple data-blocks, across which the fading amplitudes are assumed to vary. A PARAFAC-based estimator using the ALS algorithm is proposed for joint recovering of the directions of arrival, time delays and complex amplitudes of the multipaths. Simulation results are provided to demonstrate the accuracy of the proposed approach under different channel conditions.

Index Terms—Multipath channel estimation, wireless channels, MIMO systems, tensor modeling, PARAFAC decomposition.

I. INTRODUCTION

THE issue of parametric multipath channel estimation has been exploited in several works [1–3]. Most of approaches are based on subspace methods, which exploit shift-invariance properties and/or the knowledge of the pulse shape function. Simultaneous estimation of angles of arrival and delays benefits from the fact that path amplitudes are fast-varying while angles and delays are slowly-varying over multiple transmission blocks or data-blocks. In [1–3], the angles and delays are blindly-estimated using a collection of previous estimates of the space-time channel impulse response. As in [2,3], the linear-phase variation property of the frequency domain transformed version of the known pulse shape function is exploited.

Training-sequence-based space-time channel estimation methods exploiting the multiblock invariance of angles and delays have been proposed recently in [4]. In [4,5], unstructured methods are proposed which are based on the invariance properties of the spatial and temporal subspaces. In [6], a multiblock approach is proposed for multipath parameter estimation in the context of Time Division-Code Division Multiple Access (TD-CDMA) systems. In [7], multiblock processing is also considered for unstructured estimation of the low-rank space-time channels.

In the context of MIMO channels, channel estimation typically uses training/pilot symbols. An accurate channel estimation is important in coherent MIMO communication systems as well as it allows the design of efficient space-time signaling techniques that better exploit the MIMO channel. Parametric channel estimation techniques relying on a physical description of the MIMO channel (i.e. multipath angles, delays and fading amplitudes) are of great interest in wireless position-location systems and future wireless intelligent networks. Different approaches for channel estimation have been proposed in several works [8–10]. [8] proposes a modal analysis/filtering concept which exploits the different varying rates of the multipath parameters for estimating time-varying (block-fading) frequency-selective MIMO channels. The authors show that more accurate channel estimates with respect to the standard LS estimation method can be obtained. The approach proposed in [10] is based on spectral factorizations of the specular channel into stationary (space) and non-stationary (fading amplitudes) signature subspaces, and uses linear prediction for estimating/tracking the time-varying channel. In [9], a subspace method is proposed for the parametric estimation of physical MIMO channels. This approach works on a previous unstructured channel estimate, and performs a subspace decomposition of the channel covariance matrix to determine Directions Of Arrival/Departure (DOAs/DODs), and delays.

In this paper, we propose a new approach for modeling and estimation of space-time multipath MIMO channels based on tensor modeling. We make use of the fact that the variation of multipath amplitudes over multiple data-blocks is faster than that of angles and delays for showing that the received signal can be modeled as a third-order (3D) tensor. A PARAllel FACtor (PARAFAC) tensor model [11] arises thanks to the use of a training sequence which is periodically extended over multiple data-blocks, which are jointly processed at the receiver. By tapping on the powerful identifiability properties of the PARAFAC decomposition, it is possible to perform the the estimation of the complete set of MIMO multipath parameters: DOAs, DODs, delays and fading amplitudes. The estimation method consists in using the ALS algorithm, followed by a final estimation stage that relies on the knowledge of the training sequence.

Contrarily to other parametric channel estimation approaches such as [1–3], in which multipath parameters are extracted from unstructured estimations of the space-time channels, the proposed PARAFAC modeling approach directly works on the received signal, thus avoiding error propagation in cases where the unstructured space-time channel is not accurately estimated. Numerical results from computer simulations show that the PARAFAC-based estimator is capable of estimating the triplet angle-delay-amplitude for each multipath with good accuracy even for short training sequences, provided that the num-
ber of processed data-blocks is enough. The proposed estimator also performs well with fewer receiver antennas than multipaths.

The organization of this paper is as follows. Section II describes the system model and the main assumptions. The channel time-varying channel model and the training sequence structure are also presented in this section. In Section III, an overview of the PARAFAC decomposition of a third-order tensor is provided as the basis of the proposed approach. Section IV formulates the MIMO channel and the received signal using PARAFAC modeling. This section also discussed the identifiability of the proposed PARAFAC model and an algorithm for the estimation of the MIMO channel parameters is presented. In Section V, some simulation results are presented for performance evaluation. The paper is concluded in Section VI.

II. System overview

Let us consider a MIMO wireless communication system in which a digital signal is transmitted in a specular multipath environment, i.e., the channel is parameterized as the superposition of $L$ faded paths. The transmitter and the receiver are equipped with arrays of $M$ and $K$ antennas, respectively, spaced half wavelength or closer, so that we can apply the far-field approximation by assuming a locally plane wave. This model is widely used for outdoor scenarios and has been adopted for MIMO systems [8]. We focus on the case of a single-user transmission to simplify the model formulation. The generalization of the proposed model to the multiuser case is straightforward.

Figure 1 illustrates the considered MIMO propagation scenario. Each path is associated with a different scatterer located between the transmitter and the receiver. The location of the $l$-th scatterer determines a DOD $\phi_l$ and a DOA $\theta_l$ (with respect to the transmit/receive array broadside) and a relative propagation delay $\tau_l$ for the $l$-th path [12]. It is assumed that the maximum path delay exceeds the inverse of the coherence bandwidth so that the channel is frequency-selective. The finite support of the channel impulse response is equal to $I$ symbol periods and the oversampling factor at the receiver is equal to $P$ times the symbol rate. Let us define the following matrices collecting the transmitter and the receiver array responses and as well as the combined transmitter/receiver pulse shape responses:

$$
A_{tx}(\phi) = [a_{tx}(\phi_1) \cdots a_{tx}(\phi_L)] \in \mathbb{C}^{M \times L}
$$

$$
A(\theta) = [a(\theta_1) \cdots a(\theta_L)] \in \mathbb{C}^{K \times L}
$$

$$
G(\tau) = [g(\tau_1) \cdots g(\tau_L)] \in \mathbb{C}^{L \times IP}.
$$

A. Block-fading MIMO channel model

The transmitted information symbols are organized into $N_b$ data-blocks. We adopt a block-fading model for the space-time channel, which is based on the fact that, angles and delays (long-term parameters) usually experience a much slower rate of variation than the fading amplitudes (short-term parameters). We assume that the data-blocks are sufficiently short so that the channel fading can be regarded as stationary over a time-interval necessary for the transmission of a whole data-block and it varies independently from block to block. This is typically the case of Time Division Multiple Access (TDMA)-based systems. In such type of system, different data-blocks are allocated to different mobile users [4]. Therefore, we shall adopt a “block-fading model” for the time-varying propagation channel. This block-fading channel model is reasonable in most of mobile communication systems with block-transmission, and has been exploited in [8, 10], for purposes of MIMO channel estimation.

B. Multi-block training sequence

At the transmitter, each transmission block is organized in $M$ data streams that are transmitted by the $M$ transmit antennas. The structure of these data streams depend on the considered particular scheme (e.g., spatial multiplexing, space-time coding, etc). Each one of the $M$ data streams has a training sequence of $N$ symbols known at the receiver.

The length-$N$ training sequence at the $n$-th transmit antenna for the $n_b$-th transmission block is represented by:

$$
s_{m}(n_b) = [s_{m}(n_b, 1) \cdots s_{m}(n_b, N)]^T \in \mathbb{C}^N.
$$

We make the following assumptions concerning the design of the training sequences:

A.1 The $M$ training sequence vectors $s_1, \ldots, s_M$ are linearly independent;

A.2 The training sequence length $N$ satisfies $N \geq MI$;

A.3 The training sequence $s_{m}, m = 1, \ldots, M$, is reused across $N_b$ successive transmission blocks, and we have $s_{m}(n_b) = s_{m}, \forall n_b \in [1, N_b]$.

We remark that the “independence” assumption does not lead to an optimal training sequence set for estimating the MIMO channel. An optimal design should ensure that the training sequences have perfect periodic auto-correlations and cross-correlations within $I-1$ temporal shifts [13], where $I$ is the temporal span of the channel impulse response. Here, we are not concerned with optimal training sequence design, and we simply assume independent training sequences. As will be shown later in our simulation results, the independence assumption is enough to

![Fig. 1. MIMO multipath propagation scenario](image-url)
guarantee accurate estimates of the MIMO channel using the proposed approach.

Figure 2 outlines the multiblock MIMO transmission structure with training sequence reuse across transmission blocks. It is to be noted that one transmission block comprises $M$ parallel data blocks, each one of which having its own training sequence. Note also that the figure indicates that the same set of training sequences is inserted into $N_b$ transmission blocks. Each data block has $N_{\text{block}} = N + N_{\text{data}}$ symbols, $N_{\text{data}}$ denotes the number of “useful” data symbols of each data block. For signal modeling and channel estimation purposes, we focus only on the training sequence portion of each data block. After having estimated the channel, the useful data portion can be processed/recovered in a subsequent step by means of space-time processing.

III. BACKGROUND ON THE PARAFAC DECOMPOSITION

The PARAllel FACtor (PARAFAC) decomposition, also known as CANonical DECOMPosition (CANDDECOMP), was independently developed by Harshman [11] and Carol & Chang [14] in the seventies. It is also known by the acronym CP (Candecomp-Parafac). For a third-order tensor, it is a decomposition of a tensor in a sum of triple products or triads. PARAFAC can be equivalently stated as a decomposition of a three-way array in a sum of rank-1 tensors. The PARAFAC decomposition of a tensor $X \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ has the following scalar form:

$$x_{i_1, i_2, i_3} = \sum_{q=1}^{Q} a_{i_1, q} b_{i_2, q} c_{i_3, q},$$

where $a_{i_1, q} = [A]_{i_1, q}$, $b_{i_2, q} = [B]_{i_2, q}$ and $c_{i_3, q} = [C]_{i_3, q}$ are scalar components of factor matrices $A \in \mathbb{C}^{I_1 \times Q}$, $B \in \mathbb{C}^{I_2 \times Q}$ and $C \in \mathbb{C}^{I_3 \times Q}$ respectively. $Q$ is the number of factors, also known as the rank of the decomposition. The columns of the first-, second- and third-factor matrices $A$, $B$ and $C$ are respectively called first-, second- and third-mode factor loadings.

We can state the PARAFAC decomposition using a matrix-slice notation. This notation characterizes the tensor by a set of parallel matrix-slices that are obtained by “slicing” the tensor in a given “direction”. Each matrix-slice is obtained by fixing one index of a given mode and varying the two indices of the other two modes. For a third-order tensor, there are three possible slicing directions. Let us call $X_{i_1,} \in \mathbb{C}^{I_2 \times I_1}$ the $i_1$-th first-mode slice, $X_{i_2,} \in \mathbb{C}^{I_3 \times I_2}$ the $i_2$-th second-mode slice and $X_{i_3,} \in \mathbb{C}^{I_1 \times I_3}$ the $i_3$-th third-mode slice. From the three possible slicing directions of $X$, we get the following notation for the PARAFAC decomposition:

$$X_{i_1,} = BD_{i_1}(A)C^T,$$

$$X_{i_2,} = CD_{i_2}(B)A^T,$$

$$X_{i_3,} = AD_{i_3}(C)B^T,$$

where $D_{i_1}(A)$ forms a diagonal matrix holding the $i_1$-th row of $A \in \mathbb{C}^{I_1 \times Q}$ on its main diagonal.

By stacking row-wise the first-, second- and third-mode slices we have:

$$X_1 = \left[ \begin{array}{c} X_{1,1} \\ \vdots \\ X_{I_1,1} \end{array} \right] = \left[ \begin{array}{c} AD_1(C) \\ \vdots \\ BD_1(C) \end{array} \right] B^T = (C \odot A)B^T,$$

$$X_2 = \left[ \begin{array}{c} X_{1,} \\ \vdots \\ X_{I_1,} \end{array} \right] = \left[ \begin{array}{c} AD_1(C) \\ \vdots \\ BD_1(C) \end{array} \right] C^T = (A \odot B)C^T,$$

$$X_3 = \left[ \begin{array}{c} X_{1,} \\ \vdots \\ X_{I_1,} \end{array} \right] = \left[ \begin{array}{c} CD_2(B) \\ \vdots \\ CD_2(B) \end{array} \right] A^T = (B \odot C)A^T,$$

where $\odot$ denotes the Khatri-Rao (column-wise Kronecker) product.

One of the most interesting properties of PARAFAC is its uniqueness. Contrarily to bilinear (matrix) decompositions, which are in general not unique for ranks greater than one (rank-one matrices are unique up to a scalar factor), the PARAFAC decomposition of tensors of rank greater than one can be unique up to scaling and permutation of factors [11, 15].

IV. PARAFAC MODELING OF THE MIMO CHANNEL AND THE RECEIVED SIGNAL

The block-fading MIMO channel can be viewed as a fourth-order tensor $H \in \mathbb{C}^{N_s \times N \times M \times IP}$. Let us define $h_{n,k,m,i'}$ as a scalar component of the MIMO channel tensor $H$, which represents the impulse response of the $i'$-th tap of the channel between the $m$-th transmit and $k$-th receive antenna for the $n$-th fading block, and $i' = (i-1)P + p - 1$. We propose a third-order PARAFAC model for the block-fading MIMO channel. In PARAFAC form, the scalar component $h_{n,k,m,i'}$ of the $L$-path block-fading MIMO channel is a direct generalization of [16], and can be written as:

$$h_{n,k,m,i'} = \sum_{i=1}^{L} \beta_{n,k,i} \alpha_{m,j} \gamma_{l,i'},$$

where $\beta_{n,k,i}, \alpha_{m,j}$ and $\gamma_{l,i'}$ are scalar components of the factor matrices $\beta, \alpha$ and $\gamma$ respectively.
where $\beta_{n,t} = [B]{n}_{t}, \ a_{k,t} = [A(\theta)]_{k,t}, \ \pi_{m,t} = [A_{I_{2}}(\phi)]_{m,t}, \ g_{t,i} = [G(\tau)]_{t,i}$. 

After baseband conversion and oversampling at each receive antenna, we collect $NP$ received samples at each receive antenna. Let us define $x_{n,k,n'}$ as a scalar component of the received signal tensor $X \in \mathbb{C}^{N_{s} \times K \times NP}$, representing the $n'$-th received signal sample at the $k$-th antenna for the $n$-th transmission block, and $n' = (n-1)P+p-1$. In absence of noise, $x_{n,k,n'}$ can be written as:

$$x_{n,k,n'} = \sum_{m=1}^{M} \sum_{i'v=1}^{1P} h_{n,k,m,i'} s_{n',m,i'},$$

(5)

where

$$s_{n',m,i'} = [\mathbf{S}]_{n',(i'-1)M+m},$$

is an element of $\mathbf{S} = \mathbf{S} \otimes \mathbf{I}_{P} \in \mathbb{C}^{NP \times MIP}$, and

$$\mathbf{S} = \text{block-Toeplitz}(s_{1}, \ldots, s_{M}) \in \mathbb{C}^{N \times M} \quad \text{(6)}$$

is a block-Toeplitz training sequence matrix. The $n$-th slice of the received signal, denoted by $X_{n,k} \in \mathbb{C}^{K \times NP}$, can be expressed as a function of the MIMO multipath parameters as:

$$X_{n,k} = \mathbf{A}(\theta)D_{n,k}(\mathbf{B})U^{T}(\tau, \phi), \quad n_{b} = 1, \ldots, N_{b}, \quad \text{(7)}$$

where $U(\tau, \phi) \in \mathbb{C}^{L \times MIP}$ is defined as:

$$U(\tau, \phi) = \mathbf{A}(\theta) \otimes \mathbf{G}(\tau) \in \mathbb{C}^{MIP \times L} \quad \text{(8)}$$

The $n_{b}$-th matrix-slice of the received signal, denoted by $X_{n,k} \in \mathbb{C}^{K \times NP}$, can be written as:

$$X_{n,k} = \mathbf{H}_{n,k} \cdot \mathbf{S}^{T} = \mathbf{A}(\theta)D_{n,k}(\mathbf{B})\mathbf{C}(\tau, \phi), \quad n_{b} = 1, \ldots, N_{b}, \quad \text{(9)}$$

where

$$\mathbf{C}(\tau, \phi) = \mathbf{S}U(\tau, \phi) \in \mathbb{C}^{NP \times L} \quad \text{(10)}$$

is a combined space-time channel response at the receiver side, i.e., a convolution between the receiver space-time signatures and the training symbols.

Let us stack $N_{b}$ slices $X_{1}, \ldots, X_{N_{b}}$: in a matrix $X_{2} \in \mathbb{C}^{K_{N_{b}} \times NP}$, and $N_{b}$ slices $H_{1}, \ldots, H_{N_{b}}$: in a matrix $H_{2} \in \mathbb{C}^{K_{N_{b}} \times MIP}$:

$$X_{2} = \left[ \begin{array}{c} X_{1} \\ \vdots \\ X_{N_{b}} \end{array} \right], \quad H_{2} = \left[ \begin{array}{c} H_{1} \\ \vdots \\ H_{N_{b}} \end{array} \right].$$

$X_{2}$ and $H_{2}$ are unfolded representations of the tensors $X$ and $H$, respectively. Using (7)-(8), we obtain the following input-output relation:

$$X_{2} = H_{2}S^{T} = (\mathbf{B} \circ \mathbf{A}(\theta))C^{T}(\tau, \phi),$$

(11)

where

$$H_{2} = (\mathbf{B} \circ \mathbf{A}(\theta))U^{T}(\tau, \phi).$$

The two other unfolded matrix representations are:

$$X_{3} = (\mathbf{A}(\theta) \circ \mathbf{C}(\tau, \phi))B^{T} \in \mathbb{C}^{NP \times N_{b}}, \quad X_{1} = (\mathbf{C}(\tau, \phi) \circ \mathbf{B})A^{T}(\theta) \in \mathbb{C}^{N_{b} \times NP \times K}.$$

### A. Identifiability

Identifiability of (11) allows one to uniquely determine (up to trivial ambiguities) the parameters of the $L$ multipaths from the observed received signal tensor $X \in \mathbb{C}^{N_{s} \times K \times NP}$. According to the identifiability results of the PARAFAC model, the identifiability of $\mathbf{A}(\theta), \mathbf{B}$, and $\mathbf{C}(\tau, \phi)$ is linked to the concept of $k$-rank of these matrices, which is defined as follows.

**Definition ($k$-rank):** The rank of $\mathbf{A} \in \mathbb{C}^{I_{1} \times Q}$, denoted by $r_{A}$, is equal to $r$ if $\mathbf{A}$ contains at least a set of $r$ linearly independent columns but no set of $r+1$ linearly independent columns. The Kruskal-rank (or $k$-rank) of $\mathbf{A}$ is the maximum number $k$ such that every set of $k$ columns of $\mathbf{A}$ is linearly independent. Note that the $k$-rank is always less than or equal to the rank, and we have:

$$k_{\mathbf{A}} \leq r_{\mathbf{A}} \leq \min(I_{1}, Q).$$

Based on the $k$-rank concept, the following result is important to study the identifiability of our model [17, 18]:

**Theorem 1:** Consider the set of $N_{b}$ slices $X_{n,k} = \mathbf{A}(\theta)D_{n,k}(\mathbf{B})C^{T}(\tau, \phi), \quad n_{b} = 1, \ldots, N_{b}$, defined in (9). If

$$k_{\mathbf{A}(\theta)} + k_{\mathbf{B}} + k_{\mathbf{C}(\tau, \phi)} \geq 2(L+1),$$

(12)

the matrices $\mathbf{A}(\theta), \mathbf{B}$, and $\mathbf{C}(\tau, \phi)$ are unique up to permutation and scaling of columns.

It is worth mentioning that the permutation ambiguity does not need to be solved in the context of the multipath parameter estimation problem, since the ordering of multipath spatial and temporal responses is not important. Concerning the scaling ambiguity, it can be eliminated from our model by exploiting the Vandermonde structure of $\mathbf{A}(\theta)$ and $U(\tau, \phi)$.

A sufficient condition for identifying the MIMO multipath parameters can be obtained by recalling useful results on the $k$-rank of a matrix having Khatri-Rao product as well as on the $k$-rank of a Vandermonde matrix (see [19], [20] for details). A sufficient identifiability condition for our model is derived in the following theorem:

**Theorem 2:** Suppose that the $L$ multipaths have statistically independent propagation (i.e., distinct DODs, DOAs and delays). A sufficient condition for identifiability is:

$$\min(N_{b}, L) + \min(K, L) + \min(M + IP - 1, L) \geq 2(L + 1).$$

**Remark:** This identifiability condition is sufficient but not necessary. Assuming $M > 1$ and $N > 1$ (irrespective of the oversampling factor $P$), a necessary condition is $k_{\mathbf{B}} \geq 2$. In practice, this means that at least $N_{b} \geq 2$ transmission blocks must be collected at the receiver to ensure uniqueness of model (11).

### B. Estimation of the MIMO channel parameters

The estimation of the MIMO multipath parameters is done in two-stages. The first one is blind, and consists in using the trilinear Alternating Least Squares (ALS) algorithm [21] for the joint estimation of angles, delays and...
amplitudes of the multipaths. Recall from Section IV that the received signal admits three matrix representations \( \mathbf{X}_1 \in \mathbb{C}^{N_b \times NP \times K} \), \( \mathbf{X}_2 \in \mathbb{C}^{K \times N_b \times NP} \) and \( \mathbf{X}_3 \in \mathbb{C}^{NP \times K \times N_b} \). The matrices \( \mathbf{A}(\theta), \mathbf{B} \) and \( \mathbf{C}(\tau, \phi) \) are estimated by optimizing the three following least squares criteria:

\[
\begin{align*}
\arg\min_{\mathbf{B}} \| \mathbf{X}_3 - (\mathbf{A}(\theta) \odot \mathbf{C}(\tau, \phi)) \mathbf{B}^T \|_F^2, \\
\arg\min_{\mathbf{A}(\theta)} \| \mathbf{X}_1 - (\mathbf{C}(\tau, \phi) \odot \mathbf{B}) \mathbf{A}(\theta)^T \|_F^2, \\
\arg\min_{\mathbf{C}(\tau, \phi)} \| \mathbf{X}_2 - (\mathbf{B} \odot \mathbf{A}(\theta)) \mathbf{C}(\tau, \phi)^T \|_F^2,
\end{align*}
\]

where \( \mathbf{X}_i, i=1,2,3 \) are the noisy versions of \( \mathbf{X}_i, i=1,2,3 \), where \( \| \cdot \|_F \) denotes the Frobenius norm of its matrix argument. One complete iteration of the ALS has three updating steps. The basic idea is to update one factor matrix using the least squares algorithm, conditioned on previously obtained estimates for the remaining factor matrices that define the decomposition. At the end of the ALS algorithm, we will have the estimates \( \mathbf{A}(\theta), \mathbf{B}, \) and \( \mathbf{C}(\tau, \phi) \).

The second stage consists in using the training sequence matrix \( \mathbf{S} \) to find an LS estimate of \( \mathbf{U}(\tau, \phi) = \mathbf{A}_{12}(\phi) \odot \mathbf{G}(\tau) \) as:

\[
\hat{\mathbf{U}}(\tau, \phi) = \mathbf{S} \hat{\mathbf{C}}(\text{conv})(\tau, \phi).
\]

It is worth noting that separated estimations of \( \mathbf{A}_{12}(\phi) \) and \( \mathbf{G}(\tau) \) as well as the elimination of the scaling factors can be carried out by exploiting the Vandermonde structures of \( \mathbf{A}(\theta) \) and \( \mathbf{A}_{12}(\phi) \). Note that \( \mathbf{U}(\tau, \phi) \) has a Khatri-Rao factorization structure. Since the first row of \( \mathbf{A}_{12}(\phi) \) and \( \mathbf{A}(\theta) \) only have unitary entries, i.e. \( (\mathbf{A}_{12}(\phi))_{12} = [1, \ldots, 1] \) and \( (\mathbf{A}_{1}(\theta))_{1} = [1, \ldots, 1] \), an estimate of \( \mathbf{G}(\tau) \) can be extracted, for instance, from the first matrix block of \( \hat{\mathbf{U}}(\tau, \phi) \) of dimension \( L \times IP \) (c.f. (8)).

V. SIMULATION RESULTS

In this section, some simulation results are shown to illustrate the performance of the proposed parametric MIMO channel estimator. We assume \( N = 10 \) training symbols per transmit antenna. The training symbols are modulated using Binary Phase Shift Keying (BPSK). The oversampling factor is assumed to be \( P = 2 \). The pulse shape function is a raised cosine with roll-off 0.35. We consider \( L = 3 \) specular multipaths with equal average power. The vector containing the DODs, DOAs and delays of the multipaths are respectively \( \theta = [-10^\circ, 30^\circ, 50^\circ], \) \( \tau = [0, T, 2T] \), where \( T \) denotes the symbol period (\( I=3 \) is assumed). The fading amplitudes are modeled as complex Gaussian random variables and assumed to be uncorrelated between two successive blocks.

In order to evaluate the accuracy of the proposed method in estimating the spatial signatures, Fig. 3 depicts the normalized MUSIC spectrum for the DODs and DOAs. We have assumed \( M = K = 4, N_b = 10 \) and a SNR of 20dB. We can see that accurate estimates of the transmitter and receiver spatial signatures are obtained. Figure 4 shows the RMSE between estimated \( \mathbf{H} \) and true \( \mathbf{H} \) channel matrices as a function of the SNR. These results are an average over 1000 independent realizations assuming \( M = K = 2 \) and \( N_b = 3, 10 \) or 30. In this simulation, 95% of the runs were retained for plotting the results. The 5% worst (ill-convergent) runs were discarded. Convergent runs have converged within 30 iterations in average. Note that the estimation performance improves as the number of transmission blocks is increased. In fact, fading amplitudes variation across the blocks is converted into temporal diversity for resolving the multipath signals.

VI. CONCLUSION AND PERSPECTIVES

In this paper, we have presented a new method for estimating space-time wireless channels based on PARAFAC modeling. The proposed estimator relies on a parametric channel model for a time-varying multipath channel. In other words, we have used the fact that the variation of the
multipath amplitudes over multiple data-blocks is faster than that of angles and delays in order to build a third-order PARAFAC model for the channel. We have shown that the received signal can also be viewed as a third-order PARAFAC model thanks to the use of a training sequence which is periodically extended over multiple data-blocks to be jointly processed at the receiver. Using the ALS algorithm, the multipath parameters are estimated directly from the received signal tensor. The PARAFAC-based estimator provides a good estimation accuracy even for short training sequences, provided that the number of processed data-blocks is large enough. Due to the identifiability properties of the PARAFAC decomposition, the proposed estimator performs well with fewer receiver antennas than multipaths.

In a future work, we should compare the proposed modeling/estimation technique with classical ones that determine the multipath parameters using previous unstructured channel estimate [1–3, 9]. Although we have used a third-order PARAFAC approach for channel modeling/estimation, which have allowed the use of classical PARAFAC identifiability results, a fourth-order tensor modeling approach is also possible by means of the block-constrained PARAFAC model.

References


